Korean J. Chem. Eng., **21**(1), 212-220 (2004)

Mathematical Modeling of Granular Activated Carbon (GAC) Biofiltration System

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(*Received 11 February 2003 • accepted 22 July 2003*)

Abstract−In this study, a mathematical model of a fixed bed Granular Activated Carbon (GAC) biofiltration system was developed to predict the organic removal efficiency of the filter. The model consists of bulk transportation, adsorption, utilization, and biodegradation of organics. The variation of the specific surface area due to biofilm growth and the effect of filter backwash were also included in the model. The intrapellet diffusion and the diffusion of substrate in the biofilm were described by linear driving force approximation (LDFA) method. Biodegradation of organics was described by Monod kinetics. Sips adsorption isotherm was used to analyze the initial adsorption equilibrium of the system. The model showed that the organic removal efficiency of the biofilter greatly depends on the parameters related to the biological activities such as the maximum rate of substrate utilization (k_{max}) and biomass yield (Y) coefficients. Parameters such as suspended cell concentration (X_s) and decay constant (K_d) had little effects on the model simulation results. The filter backwash also had no significant impact on the performance of the biofilter.

Key words: Granular Activated Carbon (GAC), Biofilter, LDFA, Biofilm, Biodegradation, Backwashing

INTRODUCTION

Wastewater treatment and reuse is a sustainable solution to the pollution caused by wastewater discharge. Adsorption is one of important treatment methods that can be used to remove the pollutants from water and wastewater. Activated carbon adsorption has been proven to be an excellent method for removing the organic pollutants from water and wastewater [Tien, 1996; Kim et al., 2001, 2002a, b]. Major hurdles include the high cost of activated carbon and carbon regeneration. The GAC biofilter can be one of the techno-economical systems that can be used in wastewater treatment and reuse.

The biological activity on the activated carbon plays the major role in removing pollutants from water and wastewater. This effect arises from the fact that pollutants present in wastewater are adsorbed on the biofilm coated activated carbon where they are biodegraded by the microbial community present in the biofilm. Several studies on the biological activity on activated carbon in water and wastewater treatment have been carried out [Chang and Rittmann, 1987a, b; Speitel et al., 1987; Kim and Min, 1993; Tien, 1994; Ravindran et al., 1996]. These studies indicated that biological growth onto activated carbon has advantages in organic and nutrients removal. The biofiltration system can be a promising wastewater treatment method as it has dual function of adsorption and biodegradation.

Performance of a GAC biofilter entirely depends on the biological activity. The growth of different types of microorganisms in different operating conditions makes it impossible to generalize the microbial activities in a biofilter. Therefore, it is important to evaluate the biofilter in terms of its operating conditions and the characteristics of the influent feed. During the last few decades, extensive studies have been conducted to develop mathematical models that describe the process dynamics of the GAC biofiltration system. These models incorporate various assumptions for mass transport resistances, biofilm growth, biofilm biodegradation, and adsorption on activated carbon to explain the systems [Ying and Weber, 1979; Andrews and Tien, 1981; Speitel et al., 1987, Chang and Rittmann, 1987a; Kim and Min, 1993; Ravindran et al., 1996; Den and Pirbazari 2002]. These models were developed for fixed bed, fluidized bed, and completely mixed stirred tank reactor systems. All these models are capable of providing valuable information of the biological processes.

In general, the organic removal efficiency of a biofilter depends on the biological phenomena. When the biomass starts accumulating in the GAC biofilter, the organic removal efficiency increases initially and then decreases because of the reduction in the specific surface area. In addition, the GAC biofilter gets clogged with biomass. Filter backwash is provided to ease the filter bed. However the backwash phenomenon has not been properly incorporated in the previous models.

This paper aims at developing a comprehensive mathematical model of a GAC biofiltration system for wastewater treatment. The developed model is based on the fundamental framework that can describe both adsorption and biological degradation behavior of wastewater pollutants in GAC biofiltration system. The major contribution to modeling is to develop a rigorous and robust approach that includes the backwash step as well as conventional schemes. In particular, the model includes the effect of the variation of the specific surface area and the bed porosity with biofilm growth in a comprehensive manner. Model simulations are studied to investigate the effect of physical and biological parameters on the GAC

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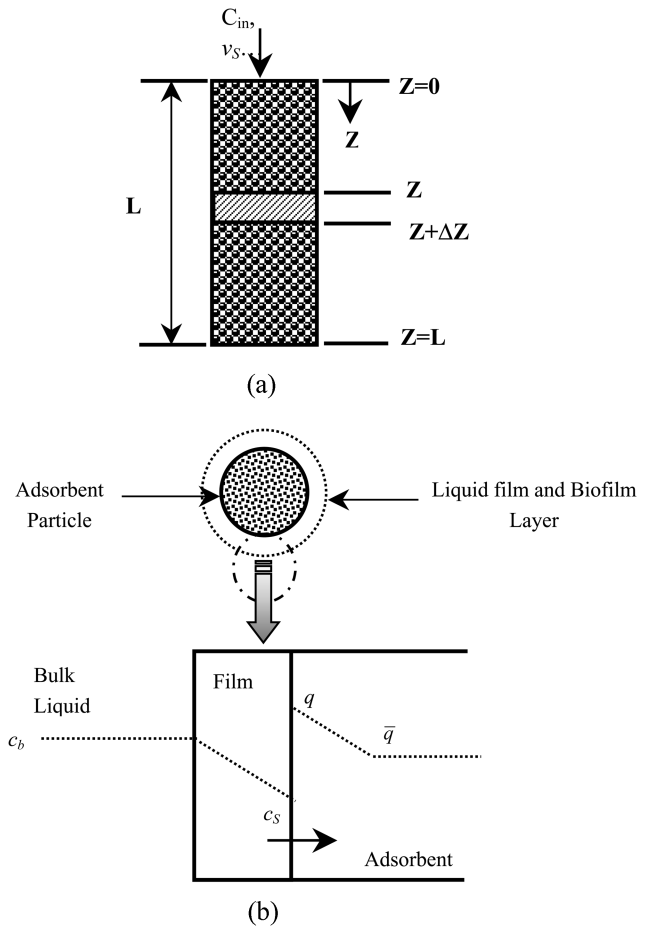

Fig. 1. Schematic representation of the mass balance in a bioactive carbon adsorber (a) and the biofilm on an adsorbent surface (b).

biofilter dynamics.

MATHEMATICAL MODEL DEVELOPMENT

A simple mathematical model was developed to evaluate the organic removal efficiency of the GAC biofiltration system. A systematic representation of the mass balance in a biologically activated carbon system and the biofilm on the activated carbon is given in Fig. 1.

The model is based on the fundamental mechanisms of transport of substrate in the bulk liquid, biofilm growth, transport and biodegradation within the biofilm, and adsorption on activated carbon. The following additional assumptions are made to simplify the modeling work.

1. The adsorbent particles are assumed to be spherical in shape and uniform in size.
2. No biological reaction occurs inside the adsorbent particle.
3. Adsorption of substrate is completely reversible.
4. The biofilm is thin relative to the radius of the adsorbent particle and can be modeled as a flat plate.
5. The biofilm is homogeneous with respect to thickness, porosity, composition, and density.
6. The curvature effect of the adsorbent surface can be ignored.
7. Any increase in biofilm thickness is due to the growth of biofilm.
8. The biological activity is assumed to be substrate limiting and is represented by the Monod equation.
9. The Glueckauf approximation may be used to describe intra-pellet diffusion.
10. The specific surface area and bed porosity vary with biofilm growth.

1. Mass Balances for Granular Activated Carbon (GAC) Biofiltration System

1-1. Substrate in the Bulk Liquid

The rates of removal of the substrate from the liquid phase by adsorption and biodegradation are given by Eq. (1):

$$\gamma_{ADS}=(1-\varepsilon_b)\cdot\frac{3N}{4\cdot\pi\cdot R_P^3},\quad \gamma_{BIO}=k_{max}\cdot\frac{C}{K_S+C}\cdot X_S \tag{1}$$

where N is the adsorbate uptake rate per pellet, ε_b is the bed porosity, R_P is the pellet radius, k_{max} is the maximum rate of substrate utilization, K_S is the Monod half velocity coefficient, X_S is the suspended cell concentration, and C is the liquid phase concentration.

The unsteady-state material balances on the substrate in the bulk liquid can be represented by the advection-diffusion equation with adsorption and reaction terms as follows [Eq. (2)]:

$$\frac{\partial C}{\partial t}=D_{ax}\cdot\frac{\partial^2 C}{\partial z^2}-u\cdot\frac{\partial C}{\partial z}-k_{max}\cdot\frac{C\cdot X_S}{K_S+C}-\frac{1-\varepsilon_b}{\varepsilon_b}\cdot a_f\cdot k_f\cdot(C-S) \tag{2}$$

where D_{ax} is the axial dispersion coefficient, u is the interstitial velocity, a_f is the specific surface area, k_f is the film mass transfer coefficient, and S is the concentration of the substrate in the biofilm.

The initial and boundary conditions are

IC: $C=C_0$

BC: z=0,

$$D_{ax}\frac{dC}{dz}=-v\cdot\left(C|_{z=0^-}-C|_{z=0^+}\right)$$

z=L,

$$\frac{dC}{dz}=0$$

1-2. Biomass Suspended in the Bulk Liquid

Suspended biomass accumulates on the adsorbent due to deposition, growth, decay, and shear loss. The equation for suspended biomass in the bulk liquid is as follows [Eq. (3)]:

$$\frac{\partial X_S}{\partial t}=\left(Y\cdot\frac{k_{max}\cdot C}{K_S+C}-K_d-\frac{\beta}{\theta\cdot\varepsilon_b}\right)\cdot X_S+\frac{1-\varepsilon_b}{\varepsilon_b}\cdot a_f\cdot X_f\cdot\sigma \tag{3}$$

where Y is the yield coefficient, K_d is the decay constant, β is the filtration efficiency, θ is the empty bed contact time, X_f is the cell density of biofilm, and σ is the biofilm shear loss coefficient.

The associated initial and boundary conditions are

I.C: $X_S=X_{S0}$

B.C: z=0, $X_S=X_{S0}$

1-3. Biofilm Diffusion and Biodegradation

Andrews and Tien [1981] proposed a conceptual model of biofilm and its growth in which they assumed that the substrate diffuses through and is taken up by the biofilm. The diffusion of the substrate across the biofilm is accompanied by its biodegradation. The model equation for biofilm diffusion with Monod type is given by Eq. (4).

$$\frac{\partial S}{\partial t} = D_f \cdot \frac{\partial^2 S}{\partial x^2} - X_f \cdot \frac{k_{max} \cdot S}{K_S + S} \tag{4}$$

where, D_f is the molecular diffusivity within biofilm.

The initial and boundary conditions are

I.C: $S = S_o$

B.C:

$x = 0$,

$$D_f \cdot \frac{\partial S}{\partial x} = \left(\frac{R_P}{3}\right) \cdot \rho_S \cdot k_P \cdot (q_S - \bar{q})$$

$x = L_f$

$$D_f \cdot \frac{\partial S}{\partial x} = k_f \cdot (C - S)$$

1-4. Biofilm Growth and Decay

Since the concentration profiles are expressed over a film thickness and the biofilm thickness varies with time, a complete description of the biofilm requires the knowledge of the film thickness as a function of time. The biofilm accumulation in the GAC biofilter due to the biological activity, deposition, decay and shear loss at each time step may be written as follows [Eq. (5)].

$$\frac{dL_f}{dt} = \int_0^{L_f} \left(\frac{Y \cdot k_{max} \cdot S}{K_S + S} - b_{tot}\right) \cdot dr \tag{5}$$

where, L_f is the biofilm thickness, and b_{tot} is the total biofilm loss coefficient.

The initial condition is

$t = 0$, $L_f = L_{fo}$

1-5. Support-phase Substrate Balance

The linear driving force approximation (LDFA) model was used to describe adsorption kinetics in this study [Eq. (6)]

$$\frac{\partial \bar{q}}{\partial t} = k_P \cdot (q_S - \bar{q}) \tag{6}$$

where, q is the adsorbed-phase concentration, $\bar{q}$ average concentration of q, q_S is the value of q at pellet surface, and k_P is the particle phase mass transfer coefficient.

1-6. Adsorption Isotherm

Several isotherms are available in the literature [Yang, 1989; Tien, 1994]. Previous studies showed that the Sips adsorption isotherm could describe the overall adsorption of organics in wastewater effectively [Shim et al., 2002]. Therefore, in this study, the Sips equation was used to analysis the biofiltration system [Eq. (7)].

$$q_S = \frac{q_m \cdot b \cdot C^{1/n}}{1 + b \cdot C^{1/n}} \tag{7}$$

where, q_m is the maximum adsorption capacity of adsorbent, b, and n are the constant in isotherm equation.

2. Bed Porosity and Specific Surface Area

The presence of biofilm outside an adsorbent results in changing the bed porosity and specific surface area.

2-1. Specific Surface Area and Bed Porosity

Alsono et al. [1997, 1998] showed that the specific surface area could be calculated based on the consideration of the area and volume of biofilm lost in each contact point as compared with no contact point between solids. Then the specific area is given by Eq. (8).

$$a_f = \frac{3 \cdot (1 - \varepsilon_{b0})}{2 \cdot R_P} \cdot \left(1 + \frac{L_f}{R_P}\right) \cdot \left[(2 - P_n) \cdot \frac{L_f}{R_P} + 2\right] \tag{8}$$

where, P_n is the number of characteristic packing spheres.

The bed porosity with biofilm, ε_b, can be calculated in the same way [Eq. (9)].

$$\varepsilon_b = 1 - (1 - \varepsilon_{b0}) \cdot \left[\left(1 + \frac{L_f}{R_P}\right)^3 - \frac{P_n}{4} \cdot \left(\frac{L_f}{R_P}\right)^2 \cdot \left(2 \cdot \frac{L_f}{R_P} + 3\right)\right] \tag{9}$$

The value of P_n has been calculated from the value of the clean bed porosity.

3. Backwashing System

It is very important to select an appropriate backwashing technique for successful operation of the GAC-biofilter. Several investigators have examined the bed expansion due to filter backwash [Lu and Huck, 1993; Tien, 1994; Ahmad and Amirtharajah, 1998]. They found no major loss of biomass during backwash of the biofilter. Serivas et al. [1991] backwashed the GAC biofilter with air scour and water routinely every 50-100 hours of continuous run, but no significant difference in vertical biomass profiles before and after backwash was observed. After backwashing, the bed length was assumed to be constant; and the specific surface area and bed porosity were calculated as follows [Eqs. (10) and (11), respectively];

$$a_f = \frac{3 \cdot (1 - \varepsilon_{b0})}{2 \cdot R_P} \cdot \left(1 + \frac{L_{f-bw}}{R_P}\right)^2 \cdot \left(1 + \frac{L_f - L_{fo}}{R_{Peff}}\right) \cdot \left[(2 - P_n) \cdot \frac{L_f - L_{fo}}{R_{Peff}} + 2\right] \tag{10}$$

$$\varepsilon_b = 1 - (1 - \varepsilon_{b0}) \cdot \left[\left(1 + \frac{L_f - L_{fo}}{R_{Peff}}\right)^3 - \frac{P_n}{4} \cdot \left(\frac{L_f - L_{fo}}{R_{Peff}}\right)^2 \cdot \left(2 \cdot \frac{L_f - L_{fo}}{R_{Peff}} + 3\right)\right] \tag{11}$$

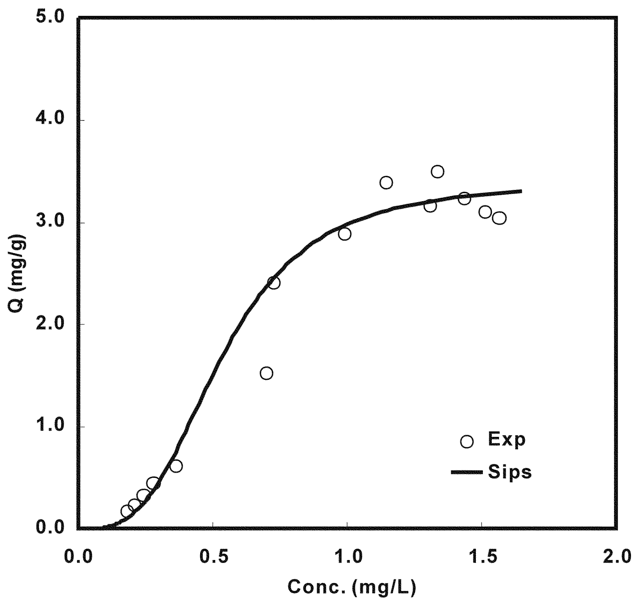

Fig. 2. Overall adsorption isotherm of the wastewater (Initial TOC concentration=3.5 mg/L) [Shim et al., 2002].

$$L_{f-max}=\frac{2}{(P_n-2)}\cdot R_{Peff},\qquad R_{Peff}=R_P+L_{fo-bw} \tag{12}$$

where, L_{f-bw} is the biofilm thickness after backwashing and R_{Peff} is the effective particle radius.

MODEL RESULTS AND DISCUSSION

The model developed in this study incorporates the mechanisms such as bulk transport, utilization, adsorption, and biofilm degradation. Model sensitivity studies could provide valuable information on the behavior of GAC biofilter as it is possible to investigate the effect of model parameters without changing their physical meaning. Therefore, it is very useful to evaluate the sensitivity analysis

Table 1. Isotherm parameters of the sips isotherm

Parameter	Values
q_m	3.394
b	7.216
n	0.314
Error (%)	5.105

Table 2. Kinetic parameters for model sensibility

Parameter	Units	Values	Parameter	Units	Values
L	m	0.04	k_{max}	s^{-1}	1.5×10^{-4}
v	m/s	2.78×10^{-4}	K_S	mg/L	0.24
ρ_p	kg/m^3	748.	D_f	m^2/s	6.8×10^{-10}
R_p	m	6.5×10^{-4}	X_S	mg/L	1.0×10^{-7}
ε_{bo}	-	0.4	L_f	m	1.0×10^{-6}
C_0	mg/*l*	1.0×10^{-7}	b_{tot}	s^{-1}	1.9×10^{-6}
k_f	m/s	1.4×10^{-5}	K_d	s^{-1}	7.9×10^{-7}
k_P	s^{-1}	5.2×10^{-5}	Y	mg/mg	0.34
			X_f	mg/L	6.4×10^{3}

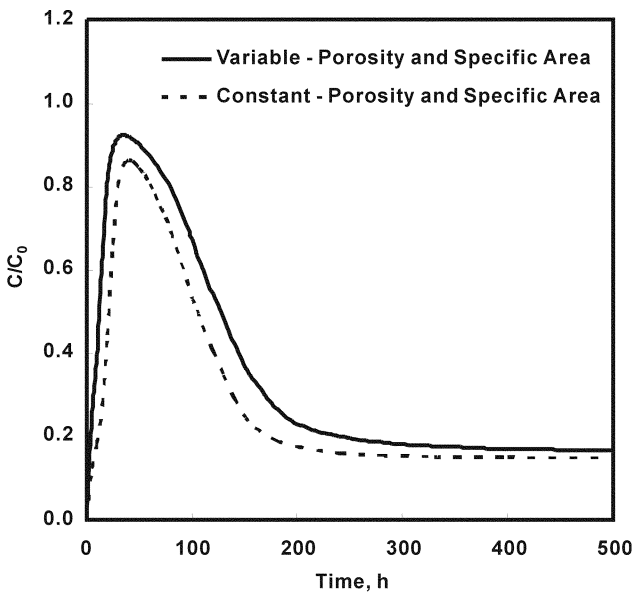

Fig. 3. Comparison of effluent concentrations for bed porosity and specific surface area.

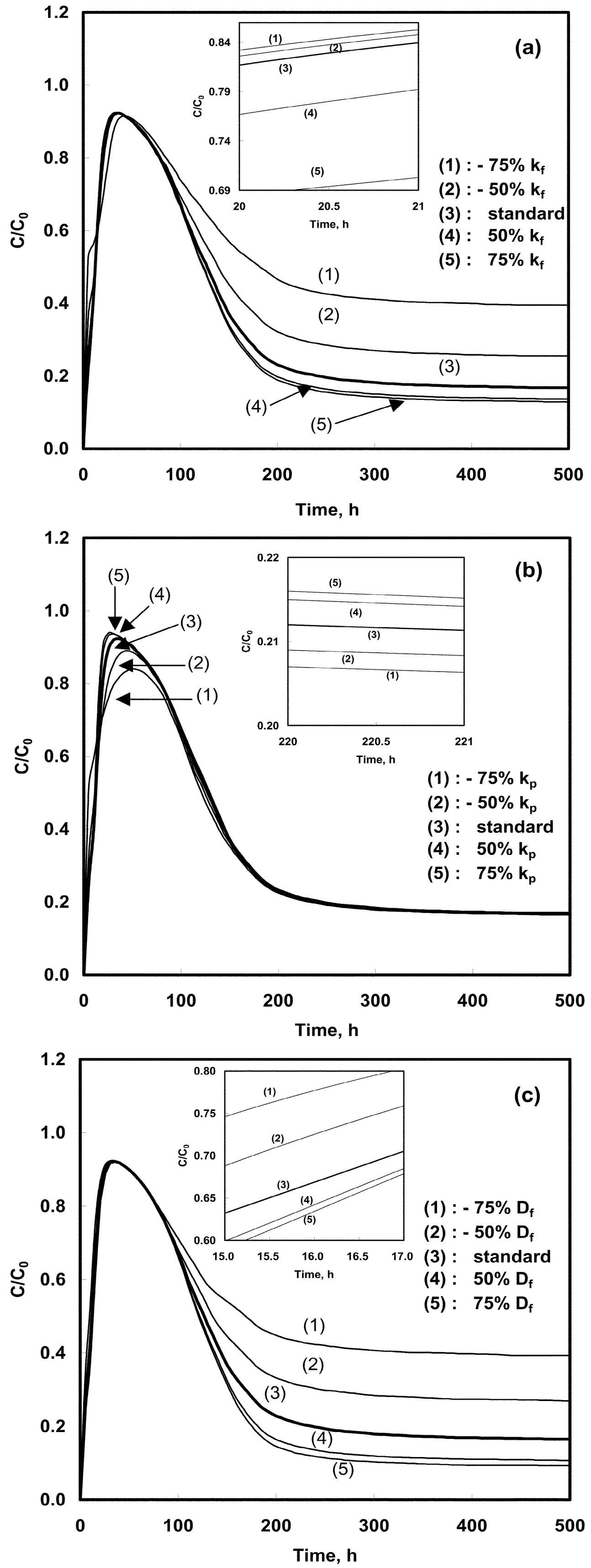

Fig. 4. Model sensitivity analysis for mass transfer coefficients, film (k_f) (a) particle (k_p) (b), and biofilm diffusion coefficient (D_f) (c).

of GAC biofilter under a various operating conditions.

A reliable set of experimental results for different initial TOC concentrations is necessary for precise estimation of the wastewater isotherm parameters. As can be seen from Fig. 2, the Sips adsorption isotherm was successful in describing the overall adsorption isotherm results of the wastewater system. The comparison of fit of data by the models is based on the mean percent deviation error, which is listed in the Table 1. The kinetic parameters obtained from the past experimental data were used in this study (Table 2) [Chang, 1987b].

The set of coupled parabolic second-order partial differential equations cannot be solved analytically. Therefore, the preferred means of numerically solving this complicated set of partial differential equations is to use the orthogonal collocation method (OCM), which can discretize the equations [Villadsen, 1967]. The partial differential equations were first reduced to a set of ordinary differential equations (ODEs) by this technique. The resulting sets of ODEs were then solved by using the subroutine DVODE [Brown, 1989]. The DVODE program employs Gear's method with variable order and step size.

The model used for simulating the dynamics of this system depends on several parameters that are mainly related to the adsorption and biodegradation phenomena.

1. Effect of Bed Porosity and Specific Area

Fig. 3 shows the comparison of effluent concentrations for bed porosity (ε_b) and specific surface area (a_f). Since the bed porosity and specific area changed with biofilm thickness, faster breakthrough was observed in the case of variable systems. As compared with the constant bed porosity and specific surface area, the organic removal efficiency of the biofilter in the case of variable system was reduced about 20%. Therefore, it is important to consider the variable bed porosity and specific area to predict the performance of the GAC biofilter system precisely.

2. Effect of Mass Transfer Coefficient

The results of sensitivity studies for the film (k_f) and solid (k_P) mass transfer coefficients are presented in Figs. 4(a) and 4(b). The effluent concentrations were dependent on the mass transfer coefficient. The solid mass transfer coefficient and the film mass transfer

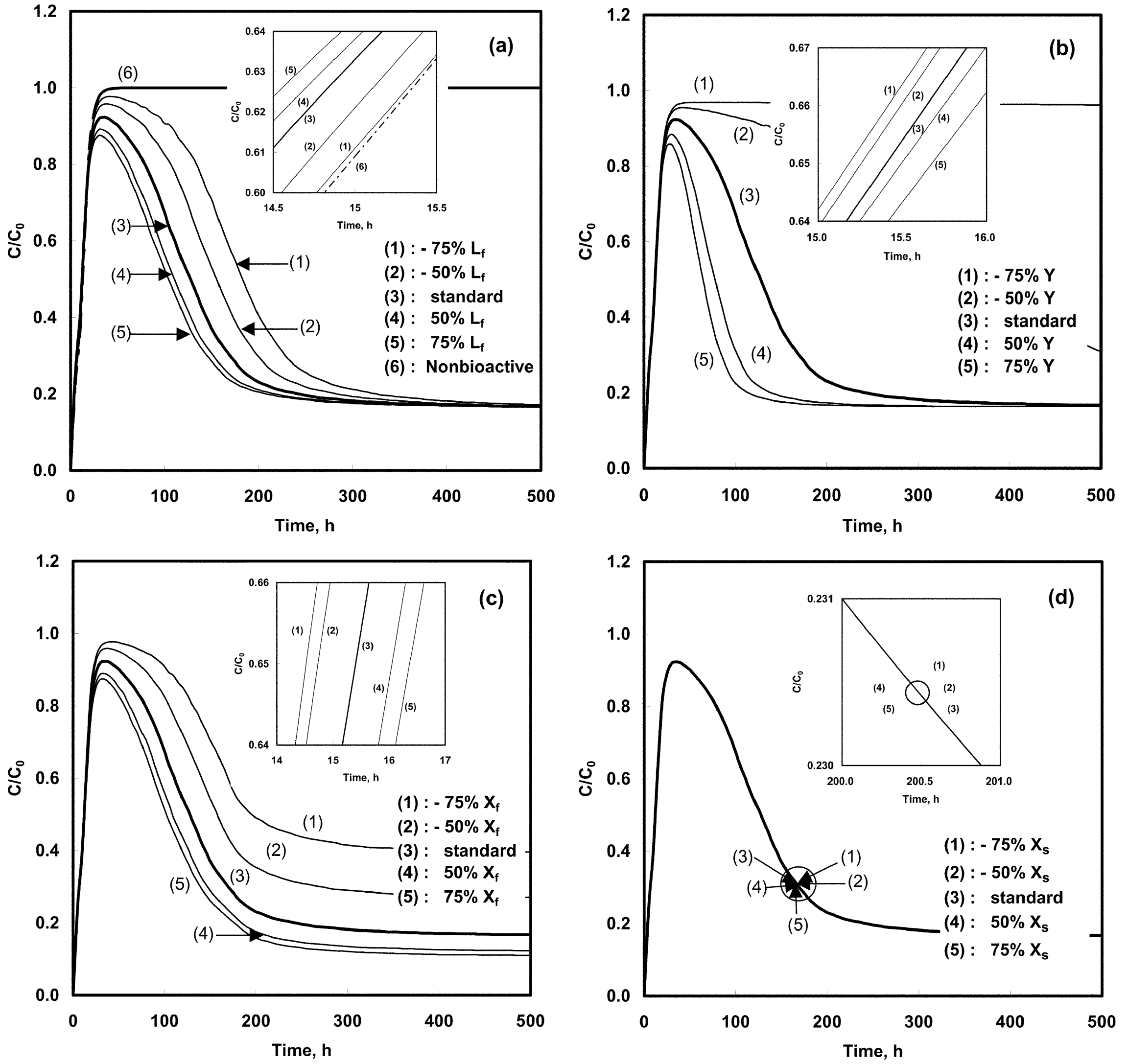

Fig. 5. Model sensitivity analysis for biofilm thickness (L_f) (a), biomass yield (Y) (b), biofilm density (X_f) (c), and suspended concentration (X_s) (d).

coefficient had no significant effect on the effluent concentration in the adsorption controlled region. The film mass transfer coefficient, however, had a profound influence on the steady state effluent concentration. The higher the film mass transfer coefficient, the greater is the organic removal efficiency. The results demonstrate that the film mass transfer parameter greatly affects the biodegradation controlled region. Fig. 4(c) also shows the effect of biofilm diffusion (D_f) on the effluent concentration. The steady state effluent concentrations were significantly dependent on biofilm diffusion. Higher biofilm diffusion can increase the organic removal efficiency.

3. Effect of Biofilm Thickness

The effect of biofilm thickness (L_f) on effluent concentration curves is shown in Fig. 5(a). Until the effluent concentration reaches a maximum, adsorption predominates in the GAC biofilter. After that, the biological phenomena controls the system till the effluent concentration profiles reach the steady state. The initial biofilm thickness increased with (i) the decrease of adsorption controlled region, and (ii) the increase of biological effect. However, the steady state effluent concentrations were not significantly affected by the change in the biofilm thickness. The presented simulation results indicate that the biofilm degradation greatly improves the process performance of the GAC biofilter. Fig. 5(b) shows the effect of biomass yield coefficient (Y) on the effluent concentration curves of the GAC biofiltration system. These results indicate that a higher yield coefficient could increase the biological effects without decreasing the adsorption effects. The steady state effluent concentrations, however, strongly depend on the biomass yield. Fig. 5(c) illustrates the effect of biofilm density (X_f) on effluent concentration profiles for GAC biofilter. The simulation results indicate that an increase in biofilm density resulted in an improved removal efficiency of the system. However, biofilm density has no significant effect in adsorp-

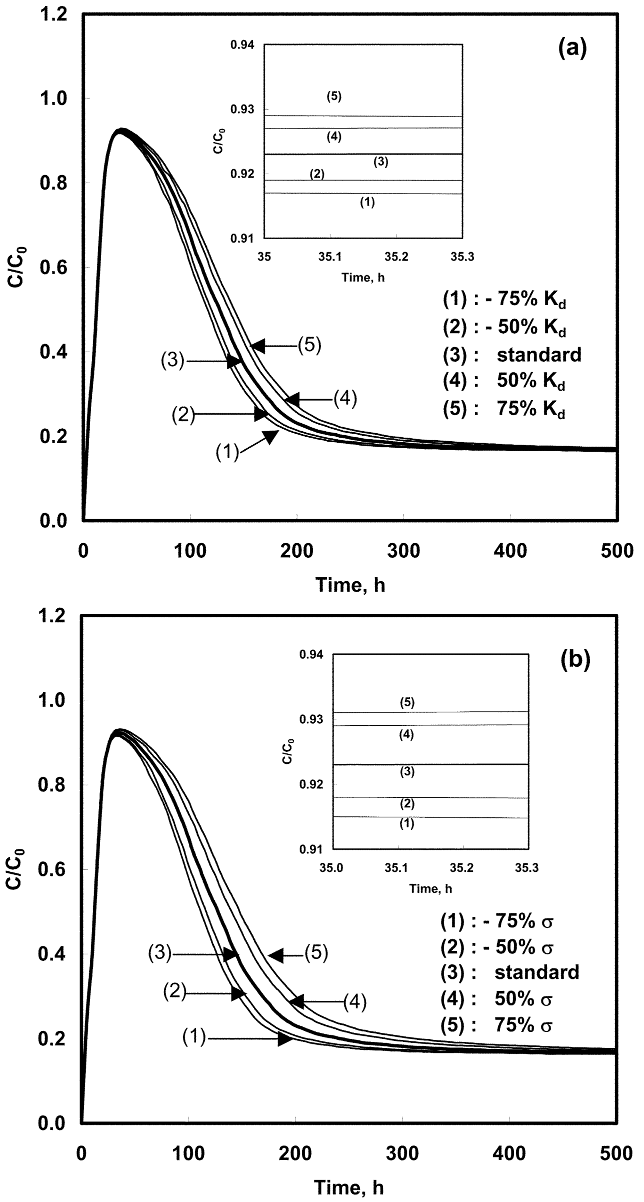

Fig. 6. Model sensitivity analysis for decay coefficient (K_d) (a) and shear loss (σ) (b).

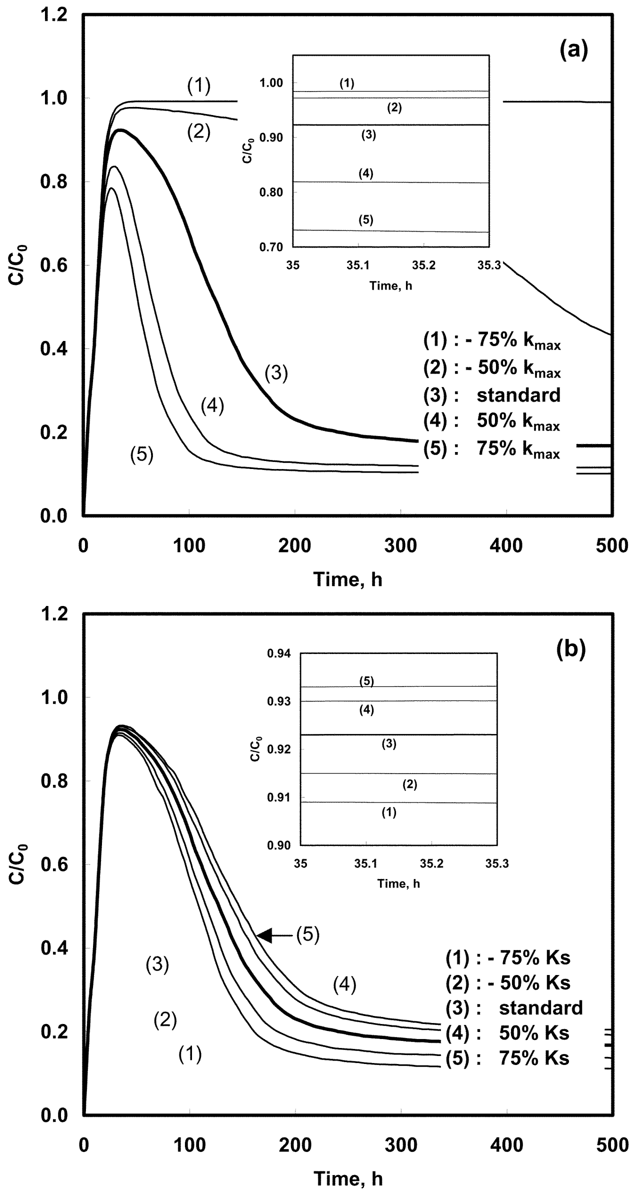

Fig. 7. Model sensitivity analysis for maximum rate of substrate utilization (K_{max}) (a) and Monod half velocity coefficient (K_s) (b).

tion capacity. The effect of variations of the concentrations in suspension (X_S) is depicted in Fig. 5(d). The effluent concentration profiles for different concentrations in suspension (±75% higher or lower than standard condition) showed the same results. It can be seen from the simulation profiles that the suspended concentrations had no significant effect on the effluent breakthrough patterns.

4. Effect of Decay Coefficient and Shear Loss

The effects of variations in total biofilm loss coefficient, namely, decay (K_d) and shear loss (σ), are illustrated in Figs. 6(a) and 6(b). The profile shows that for lower decay coefficient and shear loss, the organic removal efficiency of the GAC biofilter is better. A comparison of Figs. 6(a) and 6(b) shows that the effect of decay coefficient on the process dynamics is relatively low. However, the steady state effluent concentration profiles were not greatly affected by the variations in decay constant and shear loss within a certain range (±75% higher or lower than standard condition).

5. Effect of Monod Constants

The sensitivity of GAC biofilter systems with respect to the maximum specific rate of substrate utilization (k_{max}) and half velocity concentration (K_S) is shown in Figs. 7(a) and 7(b). The simulation results were significantly sensitive to changes in the maximum specific rate of substrate utilization. As shown in Fig. 7, substrate utilization was more influential than the half velocity concentration parameter. The GAC biofilter systems showed better organic removal for higher substrate utilization and lower half velocity concentrations.

6. Effect of Backwashing

Regular backwashing with water was adopted to avoid physical clogging of the GAC biofilter. Fig. 8 shows the performance of the GAC biofilter with backwashing using water fluidization every day (5 min) of continuous run. There is no significant difference in concentration profile before and after backwash. The organic removal efficiency of the system remained constant after about 300 h of continuous run.

Fit value [Hozalski and Bouwer, 2001] was used to investigate the sensitivity analysis. To find the fit value for each parameter, the equations were defined as follows:

$$\text{Fit value} = \frac{1}{2}\left[\sum_{time\ points}\left(\frac{C(0\%) - C(\pm 50\%)}{C(0\%)}\right)^2\right] \quad (13)$$

Here, C(0%) is the effluent concentration that was related to the standard parameters, C(±50%) is the effluent concentration, the increase or decrease of standard parameters by ±50%. The results of the sensitivity analysis of GAC-biofilter are summarized in Table 3. Using the fit value, it is possible to make a quantitative comparison of the agreement between a simulation data. The sensitivity of the system increased with the increase in the fit value. The calculation results clearly showed that the substrate utilization and biomass yield coefficients were the most sensitive to the GAC biofilter system.

Therefore, in order to improve the organic removal efficiency of the GAC biofilter, it is important to investigate the effect of the substrate utilization and biomass yield coefficients thoroughly.

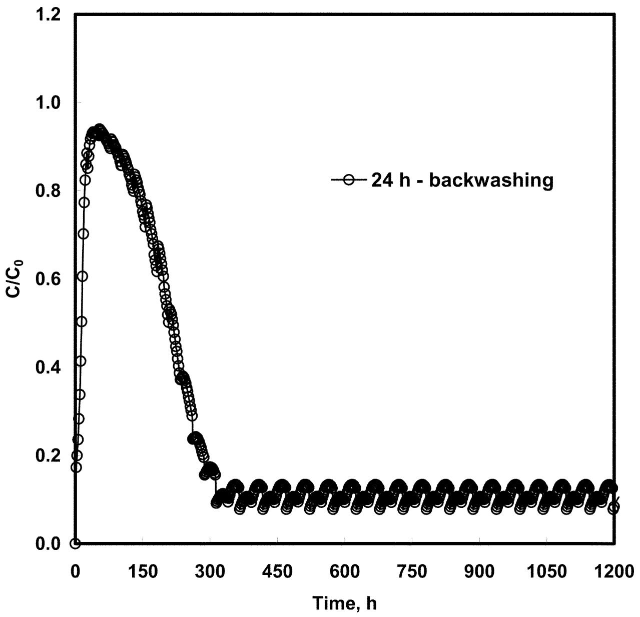

Fig. 8. Simulated performance profile from a GAC biofilter backwashed at 24 h intervals.

Table 3. Sensitivity analysis of GAC biofilter model

Parameter	Adjustment	Fit value	Average
k_{max}	+50%	0.083	1.406
	−50%	2.729	
Y	+50%	0.032	0.797
	−50%	1.561	
D_f	+50%	0.306	0.210
	−50%	0.114	
X_f	+50%	0.026	0.076
	−50%	0.126	
k_f	+50%	0.010	0.045
	−50%	0.079	
K_S	+50%	0.011	0.013
	−50%	0.015	
L_f	+50%	0.004	0.010
	−50%	0.016	
σ	+50%	4.868×10^{-3}	3.821×10^{-3}
	−50%	2.774×10^{-3}	
K_d	+50%	2.163×10^{-3}	1.777×10^{-3}
	−50%	1.392×10^{-3}	
k_P	+50%	1.993×10^{-4}	5.236×10^{-4}
	−50%	8.479×10^{-4}	
X_S	+50%	2.612×10^{-4}	1.306×10^{-4}
	−50%	0.000	

CONCLUSIONS

A simple mathematical model of a GAC biofilter was developed and the effects of several physical and biological parameters on the model prediction were investigated.

In general, the organic removal efficiency of GAC biofilter depends on the biomass. When the biomass starts to accumulate on the filter, the fixed bed characteristics such as the specific surface area and bed porosity are changed. Therefore, it is important to consider the variable bed porosity and specific area situations to model the performance of the GAC biofilter system precisely.

Model sensitivity studies showed that the effluent concentration profiles are greatly dependent on the maximum rate of substrate utilization (k_{max}) and biomass yield (Y) coefficients, while variation of some parameters such as suspended cell concentration (X_s) and decay constant (K_d) did not significantly impact the model simulation results. It was also useful to consider the effect of backwashing to simulate the operation of biofilter in practice. The filter backwash, however, had no significant impact on the performance of the biofilter.

ACKNOWLEDGMENTS

This work was supported by ARC (Australia) - KOSEF (Korea) joint research fund.

NOMENCLATURE

- a_f : specific surface area [m^2]
- b : sips equilibrium constant [L/mg]
- b_{tot} : total biofilm loss coefficient [s^{-1}]
- C : liquid phase concentration [mg/L]
- C_0 : initial concentration [mg/L]
- D_{ax} : axial dispersion coefficient [m^2/s]
- D_f : molecular diffusivity within biofilm [m^2/s]
- k_f : interphase mass-transfer coefficient from liquid to biofilm [m/s]
- k_P : solid phase mass-transfer coefficient [s^{-1}]
- k_{max} : maximum rate of substrate utilization [s^{-1}]
- K_d : Decay constant [s^{-1}]
- K_S : Monod half velocity coefficient [mg/L]
- L_f : biofilm thickness [m]
- n : Sips equilibrium constant [-]
- N : substrate uptake rate by pellet [g/s]
- q : adsorbed-phase concentration [mg/g]
- $\bar{q}$: average concentration of [mg/g]
- q_m : maximum adsorbed-phase substrate concentration [mg/g]
- q_S : value of q at pellet surface [mg/g]
- R_P : pellet radius [m]
- S : concentration of the substrate in the biofilm [mg/L]
- t : time [s]
- u : interstitial velocity [m/s]
- x : radial distance [m]
- X_f : cell density of biofilm [mg/L]
- X_S : suspended cell concentration [mg/L]
- Y : yield coefficient [mg/mg]
- z : axial distance [m]
- Y : yield coefficient [mg/mg]

Greek Letters

- β : filtration efficiency [-]
- ε_b : bed porosity [-]
- γ_{ADS} : rate of removal of the substrate from the liquid phase by adsorption defined by Eq. (1)
- γ_{BIO} : rate of removal of the substrate from the liquid phase by biodegradation defined by Eq. (1)
- θ : empty bed contact time [s]
- ρ_p : solid density [kg/m^3]
- σ : biofilm shear loss coefficient [s^{-1}]

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